

Influence of Mathematical Models in Finance on Practice: Past, Present and Future [and Discussion]

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Influence of mathematical models in finance on practice: past, present and future

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The mathematics of finance contains some of the most beautiful applications of probability and optimization theory. Yet despite its seemingly abstruse mathematics, finance theory over the last two decades has found its way into the mainstream of finance practice. Today much of the applied financial research on the use of mathematical models takes place within financial institutions. It was not always thus. The scientific breakthroughs in financial modelling both shaped and were shaped by the extraordinary flow of financial innovation which coincided with revolutionary changes in the structure of world financial markets and institutions during the past two decades. The paper covers that development with a focus on the future role of mathematical models in finance practice.

1. Introduction

The core of finance theory is the study of the behaviour of economic agents in allocating and deploying their resources, both spatially and across time, in an uncertain environment. Time and uncertainty are the central elements that influence financial behaviour. The complexity of their interaction brings intrinsic excitement to the study of finance as it often requires sophisticated analytical tools to capture the effects of this interaction. Indeed, the mathematical models of modern finance contain some of the most beautiful applications of probability and optimization theory. But of course, all that is beautiful in science need not also be practical; and surely, not all that is practical in science is beautiful. Here we have both. With all their seemingly abstruse mathematical complexity, the models of finance theory have nevertheless had a direct and significant influence on finance practice. Although not unique, this conjoining of intrinsic intellectual interest with extrinsic application is a prevailing theme of research in modern finance.

Although I intend to exemplify this theme, my remarks will not focus on deriving new practical applications of financial models, nor is it my aim to introduce any mathematical tools heretofore unknown to finance which might help to break new ground in the theory. In these directions, the practitioner, the general scientist and the mathematical finance specialist will happily find their cups abundantly filled by the multifarious papers and discussion in this Volume. Instead, in this salutatory paper, I try my hand at providing a frame of reference for what is to follow by describing something of the interaction between mathematical models and finance practice; in the past, in the present and, most importantly, in the impending future.

The origins of much of the mathematics in the papers set forth in the meeting can be traced to Louis Bachelier's (1900) magnificent dissertation on the theory of

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speculation. (See Cootner (1964) for an English translation by A. J. Boness. Mandelbrot (1989) provides a brief biography.) Completed at the Sorbonne in 1900, this work marks the twin births of both the continuous-time mathematics of stochastic processes and the continuous-time economics of option pricing. In analysing the problem of option pricing, Bachelier provides two different derivations of the Fourier partial differential equation as the equation for the probability density of what is now known as a Wiener process/brownian motion. In one of the derivations, he writes what is now commonly called the Chapman–Kolmogorov convolution probability integral, which is surely among the earlier appearances of that integral in print. In the other derivation, he takes the limit of a discrete-time binomial process to derive the continuous-time transition probabilities. (This same approach using the binomial process is now applied as a numerical approximation method to solve complicated derivative-security pricing problems; see Merton (1992*b*) for references.) Along the way, Bachelier also developed essentially the method of images (reflection) to solve for the probability function of a diffusion process with an absorbing barrier. This all took place five years before Einstein's discovery of these same equations in his famous mathematical theory of brownian motion. Not a bad performance for a thesis on which the first reader, Henri Poincaré, gave less than a top mark. (Bernstein (1992, p. 19) reports that the thesis was awarded 'mention honorable' instead of 'mention très honorable'. See Dimand (1993).)

Just as in Bachelier's option-pricing model of 1900, so the normal and binomial distributions are central to today's mathematical models of finance. Although probably coincidental, the choice of subject matter makes it altogether fitting that this Discussion Meeting be held this very week, for the historical precedent took place almost exactly 260 years ago on 12 November 1733. Abraham de Moivre, a long-time Fellow of the Royal Society, issued a short pamphlet in Latin containing the first derivation of what we now call the normal (or gaussian) distribution, as a limit to the binomial process. (As de Moivre (1738) later described the pamphlet in its English translation, 'printed November 12 1733, communicated to some Friends, but never yet made public, reserving to myself the right of enlarging my own thoughts, as occasion shall require.' Stephen Stigler, University of Chicago (personal communication). Stigler also notes that de Moivre published the first version of his *Doctrine of Chances* in *Phil. Trans. R. Soc. Lond.* **27**, 213–264 (1711).)

2. Mathematical models in practice: 1960s and before

Sophisticated mathematical models and a strong influence on practice were not always hallmarks of finance theory. Indeed, Bachelier's work was unknown in the finance literature for more than half a century. (The rediscovery of his work by economists in the early 1950s is generally credited to P. A. Samuelson (1965*a*) via the statistician, L. J. Savage.) During most of this period, finance was almost entirely a descriptive discipline with a focus on institutional and legal matters. Finance theory was little more than a collection of anecdotes, rules of thumb, and shufflings of accounting data. Mathematical models of finance were focused on the time value of money and the most sophisticated tool of analysis was present value. Application of these models by practitioners in non-financial firms was largely confined to staff who set guidelines for capital-budgeting decisions. Although the mathematical modelling of bond-price sensitivity to interest rates (*duration*) had been developed by Macaulay

in 1938, there was little evidence of its use in practice more than 20 years later, even by issuer and trader specialists in the debt markets. (Unlike in finance at this time, the mathematical models of the actuarial discipline addressed both the time value of money and the evaluation of uncertainty. The term ‘actuary’ appears to have been used first as the title for the chief executive of the Equitable Life Assurance Society at its founding in 1762. Over the next 200 years, applications of actuarial modelling involve almost exclusively evaluating mortality risks for life insurance and annuities and casualty risks for non-financial assets. For an overview of the use of actuarial methods in finance, see O’Brien 1992.)

Modern finance begins only in the late 1950s and 1960s. There were important breakthroughs in corporate finance, but the major strides in mathematical modelling came in the area of investments and capital markets. The Markowitz (1952, 1959) mean variance theory of portfolio selection provided a tractable model for quantifying the risk-return tradeoff for general assets with correlated returns. Building on Markowitz’s fundamental work, Sharpe (1964) and Lintner (1965) investigated the equilibrium structure of asset prices, and their Capital Asset Pricing Model (CAPM) became the foundational quantitative model for measuring the risk of a security. The CAPM would later form the basis for developing an entire industry to measure the investment performance of professional money managers.

Another important influence of 1960s research on investment practice was the Samuelson (1965*b*)–Fama (1965) efficient markets hypothesis which holds that in a well-functioning and informed capital market, asset-price dynamics are described by a (sub)martingale in which the best estimate of an asset’s future price is the current price, adjusted for a ‘fair’ expected rate of return. Under this hypothesis, attempts to use past price data or publicly available forecasts about future economic fundamentals to predict future security prices are doomed to failure. Earlier, Maurice Kendall (1953) of Royal Statistical Society fame had presented empirical evidence that stock and other speculative price changes were not forecastable. Indeed, Bachelier (1900) had formulated his theoretical option-pricing model assuming a martingale-like behaviour for security prices. It was, however, the Fisher & Lorie (1964) study of historical stock returns that probably first drew practitioner attention to the efficient market hypothesis. Using the newly created data base of the Chicago Center for Research in Security Prices, Fisher & Lorie showed that a randomly selected stock held from the mid-1920s to the mid-1960s would have earned, on average, a 9.4% annual compound return. Returns of this magnitude were believed to be considerably larger than those most professional managers earned for their clients during that period. Rigorous scientific confirmation of this belief was provided by a host of empirical performance studies along lines set by Jensen (1968) who used the CAPM as a benchmark to test for superior performance among United States mutual funds in the postwar period.

3. Mathematical models in practice: 1970s and 1980s

In the late 1960s and early 1970s, models of finance being developed in academe became considerably more sophisticated, involving both the intertemporal and uncertainty aspects of valuation and optimal financial decision making. Dynamic portfolio theory extended and enriched the static Markowitz mean variance model. Intertemporal and international capital asset pricing models expanded the single risk measure, beta, of the Sharpe–Lintner CAPM to multi-dimensional measures of a

security's risk. (For references and analysis on dynamic portfolio theory and intertemporal capital asset pricing, see Merton (1992*b*). Solnik (1974) was the first to adopt these models into an international framework. Although static in its formal development, the arbitrage pricing theory of Ross (1976) also provides for multiple dimensions in the measure of a security's risk.) The mathematical tools used in these models – stochastic differential and integral equations, stochastic dynamic programming, and partial differential equations – were a quantum level more complex than had been used previously.

The most important development in terms of impact on practice was the Black–Scholes model for option pricing (Black & Scholes 1973). (For the story of how they developed their model see Bernstein (1992, ch. 11); for extensive references and extensions of the model see Merton (1992*b*).) Virtually from the day it was published, this work brought the field to closure on the subject. The Chicago Board Options Exchange (CBOE) began trading the first listed options in the United States in April 1973, a month before the official publication of the Black–Scholes model. By 1975, traders on the CBOE were using the model to both price and hedge their options positions. Indeed, Texas Instruments created a hand-held calculator which was specially programmed to produce Black–Scholes option prices and hedge ratios. Such a complete and rapid adoption of finance theory into finance practice was unprecedented, especially for a mathematical model developed entirely in theory. That rapid adoption was all the more surprising, as the mathematics used in the model were not part of the standard mathematical training of either academic or practitioner economists.

The basic insight underlying the Black–Scholes model is that a dynamic portfolio trading strategy in the stock can be found which will replicate the returns from an option on that stock. Hence, to avoid arbitrage opportunities, the option price must always equal the value of this replicating portfolio. The resulting pricing formula has only one input which is not directly observable: the volatility of the return on the stock. Just as the CAPM stimulated the development of estimation techniques to measure beta, so the widespread application of the Black–Scholes model created a need for techniques to estimate standard deviations of security returns. In turn, the demand for such estimates in practice created substantial business opportunities and indeed, practitioner-developed models for estimating volatility are among the most sophisticated ones now available.

Black and Scholes, along with others, recognized that their replicating-portfolio approach could be applied to the pricing of general derivative securities with arbitrary nonlinear pay-offs contingent on one or more traded-security prices. Hence, at the same time that their work was closing gates on fundamental research on options, it was simultaneously opening new gates by setting the foundation for a new branch of finance called contingent-claims analysis (CCA). The applications of CCA range from the pricing of complex financial securities to the evaluation of corporate capital budgeting and strategic decisions and include, for instance, a unified theory for pricing corporate liabilities and the evaluation of loan guarantees and deposit insurance. Indeed, the theory and mathematical modelling of CCA for these applications have become even more important to finance practice than the original options applications.

The applications of mathematical models in the 1970s had been primarily in equity markets and equity derivative securities. The big, new applications of the 1980s were in the fixed-income arena. The models incorporated major multivariate extensions of

the CCA methodology to price and hedge virtually every kind of derivative instrument, whether contingent on equities, fixed-income securities, currencies, or commodities. Dynamic models of interest rates were combined with CCA models to price both cash-market and derivative securities simultaneously. The enormous U.S. national mortgage market could not have functioned effectively without mathematical models for pricing and hedging mortgages and mortgage-backed securities whose valuations are especially complex because of pre-payment options. By the late 1980s, the time-lag for adoption of finance theory models into practice was essentially nil. Indeed, the mathematical models used in finance practice became as sophisticated as any found in academic financial research.

The users of mathematical financial models in the 1970s were U.S. institutional equity investors, market makers and brokers trading U.S. equity options, currency traders, and a few fixed-income traders. During the 1980s, the user base of these models expanded greatly, becoming global in scope and including commercial and investment banks and institutional investors of all types (see Grundfest 1990). Practitioners in financial institutions actually took on a major role in applied research, including the creation of proprietary data bases, development of new numerical methods for solving partial differential equations, and implementation of sophisticated estimation techniques for measuring model parameters.

There are several explanations for the vastly different adoption rates of mathematical models into finance practice during the 1960s, 1970s, and 1980s. Money and capital markets in the United States exhibited historically low volatility in the 1960s: the stock market rose steadily, interest rates were relatively stable, and exchange rates were fixed. Such 'simple' market environments provided little incentive for investors to adopt new financial technology. In sharp contrast, the 1970s experienced several events that led both to structural changes and increasing volatility in financial markets. The more important of these were: the shift from fixed to floating currency exchange rates with the collapse of the Bretton Woods Agreement and the devaluation of the dollar; the world oil price crisis resulting from the creation of the Middle East cartel; the decline in the U.S. stock market in 1973/74, which was larger in real terms than any comparable period in the Great Depression; and the arrival of double-digit inflation and interest rates in the United States. In this environment, the old rules of thumb and simple regression models based on extrapolation of historical relations were wholly inadequate for making investment decisions and managing risk exposure.

During the 1970s, derivative-security exchanges were created to trade listed options on stocks, futures on major currencies, and futures on U.S. Treasury bills and bonds. The success of these markets measured in terms of trading volume can be attributed in good part to the increased demand for managing risks in the volatile economic environment. This success in turn strongly affected the speed of adoption of quantitative financial models. For example, experienced traders in the preceding over-the-counter (OTC) dealer market had achieved a degree of success by using heuristic rules for valuing options and judging risk exposures. However, these rules of thumb were soon to be found to be inadequate for trading in the fast-paced exchange-listed options market with its smaller price spreads, larger trading volume and requirements for rapid trading decisions while monitoring prices in both the stock and option markets. In contrast, formal mathematical models along the lines of the Black-Scholes model were ideally suited for application in this new trading environment.

The growth in sophistication about mathematical models and their adoption into finance practice during the 1980s far exceeded the 1970s and ran in parallel with the extraordinary growth of financial innovation (Finnerty 1992; Miller 1986, 1992). A wave of deregulation in the financial sector as well as huge government budget deficits, especially in the U.S., which increased several times the amount of sovereign debt worldwide that required intermediation, and placement were important factors driving innovation (see Miller 1986, 1992; Merton 1992*a*). They were not, however, the only ones.

Conceptual breakthroughs in finance theory in the 1980s were perhaps fewer and less fundamental than in the 1960s and 1970s, but the research resources devoted to the development and refinement of mathematical models and financial data bases to support them were considerably larger. Moreover, the opportunities and feasibility of implementing these models in practice were also much greater. (See Eckl *et al.* (1990), Marshall & Bansal (1992), and Smith & Smithson (1990) on the development of financial engineering in this period.) Major developments in computing and telecommunications technologies (including the personal computer and the orders-of-magnitude increases in computer speed and memory size) made possible the formation of many new financial markets and substantial expansions in the size of existing ones. Those same technologies made feasible the numerical solution of new complex CCA models with multivariate partial differential equations. They also speeded up the solution of existing CCA models to allow virtually real-time calculations of prices and hedge ratios. CCA and related topics were widely incorporated in top business school curricula during the late 1970s and early 1980s. As a result, by the middle 1980s there was a significant pool of MBA and Ph.D. professionals trained in modern finance theory who were available to put the theory into practice. The pool was further augmented by mathematicians and scientists with advanced degrees attracted to the financial services industry by high salaries and challenging problems.

Success of the new trading markets and intermediated products can itself lead to further success though a process called the 'financial-innovation spiral' (see Merton 1992*a, b*, 1993) that proceeds as follows: The proliferation of new trading markets in standardized securities such as futures, options, and swaps makes possible the creation of a wide range of new financial products, many custom designed and sold OTC by financial intermediaries to meet selected needs of investors and corporate issuers. Next, volume in the new markets expands further as the intermediaries themselves trade simply to hedge their own exposures from the products they sold. Such increased volume in turn reduces marginal transactions costs and thereby makes possible the further implementation of new products and trading strategies and this, in turn, leads to still more volume. New markets also evolve as some successful products become standardized and their source of distribution moves from intermediaries to markets. Success of these trading markets and custom products then encourages investment in creating additional markets and products, and so on it goes, spiralling toward the theoretically limiting case of complete markets and zero marginal transactions costs.

The reduction in transactions costs for financial institutions was substantial during the 1980s. The costs of implementing financial strategies for institutions using derivative securities such as futures or swaps are often one tenth to one twentieth the cost of using the underlying cash-market securities. This is especially the case for investments by foreign institutions which are often subject to withholding taxes on

either interest or dividends. (See Perold (1992) for an excellent description of the comparative costs of various alternatives for executing basic investment strategies. Although institutions have experienced the greatest reduction in the cost of direct market transactions, retail investors have in places had similar experiences: discount retail brokers in the U.S. now charge as little as 3–5 cents a share commissions on stock transactions.)

The decline in costs does not derive only from reductions in bid–ask spreads and commissions. There are also cost savings from movement down the learning curve. With the cumulative experience of having built several new markets, innovators become increasingly more efficient and the marginal cost of creating additional markets falls.

The same learning-curve effect applies to the application of mathematical models. Beginning in the late 1980s and continuing to the current time, the volume of derivative securities business has shifted substantially from exchange-traded derivatives to more-customized OTC contracts issued by financial institutions directly to their customers. I believe that this shift reflects a growing confidence by institutions in their valuation models that comes not only from technical improvements in the models but also from greater experience in their use. When a firm operates in exchange-traded derivatives, it has current and historical prices to calibrate its valuation model. Competition among transactors in a centralized market provides at least some protection against losses from errors in any one transactor's valuations. In a bilateral OTC transaction, the firm must rely on its valuation models without the benefit of market verification. Hence, without sufficiently reliable mathematical models for valuation, much of the financial innovation that originates in the OTC market could not take place. At the same time, such innovation increases the demand for more sophisticated models. Thus, we see that mathematical modelling both shapes and is shaped by the flow of financial innovation.

4. Mathematical models in practice: 1990s and beyond

Some see the extraordinary growth in derivative securities over the past five years as only a fad. However, a more likely explanation is the vast savings in transactions costs from their use. Looking to the future, with such cost savings we are not going back: derivatives are a permanent part of the mainstream global financial system.

It may be difficult to believe that the pace of general financial innovation during the past decade can sustain itself into the future. However, there are reasons to believe that it can. The decision to implement an innovation involves a trade-off between its benefit and the cost. With secularly declining transactions and learning-curve costs, the threshold benefit required to warrant implementation also declines secularly. Hence, holding fixed the same pace of change in the underlying economic fundamentals as in the past, the implementation of financial innovation is likely to be more rapid since the threshold for change is lower.

With much lower costs of change, it becomes profitable not only to introduce new products and create new markets, but also to change entire institutional arrangements (including geographical and political locations) in response to much smaller shifts in customer tastes or operating costs than in the past. Lower transactions costs, together with the prospect of greater global competition in financial services, form the basis for forecasting substantial increases in both the

frequency and the magnitude of institutional changes for both private sector and government financial intermediaries and for regulatory bodies. In the past, mathematical models played a key role in supporting the creation of new products and markets. In the future, that role will expand to include supporting the creation of entire new institutions.

A successful conceptual framework for analysing issues involving the global financial system in the future must address endogenously differences in institutional structure across geopolitical boundaries and in the dynamics of institutional change. The neoclassical-economics perspective addresses the dynamics of prices and quantities. But it is largely an 'institution-free' perspective in which only functions matter. It thus has nothing to say directly about cross-sectional or intertemporal differences in the institutions that serve these functions. In contrast, there is the institutional perspective in which institutions not only matter but are the conceptual 'anchor'. This perspective takes as given the existing institutional structure and views the objective of public policy as helping the institutions currently in place to survive and flourish. Framed in terms of the banks or the insurance companies, managerial objectives are similarly posed in terms of what can be done to make those institutions perform their particular financial services more efficiently and profitably. The institutional perspective addresses cross-sectional differences across borders but is static in focus. Hence, institutional change is exogenous within this perspective.

Drawing on both these perspectives, the functional perspective (see Merton 1992*a*, 1993) takes as given the economic functions performed by financial institutions and seeks to discover what is the best institutional structure for performing those functions at a given time and a given place. It does not posit that existing institutions, whether private sector or governmental, operating or regulatory, will be preserved. Functions thus serve as the conceptual anchor here. Because institutions matter but are not the anchors, institutional changes are endogenous within this perspective.

To illustrate the differences between the institutional and functional perspectives, consider a hypothetical example involving municipal-bond insurance (Merton 1993, pp. 28–29). In the U.S., there are specialized insurance companies that sell insurance contracts which guarantee interest and principal payments on municipal bonds against default by the issuer. The policies are typically sold to the issuer who attaches them to the bonds to give them the highest (AAA) credit rating. To succeed as a guarantor, the insurance company itself must be seen as a very strong credit. In evaluating the firm's competitive standing, a manager with an institutional perspective would focus on other insurance companies as competitors. A manager with a functional perspective would instead focus on the best institutional structure to perform the function, which may not be an insurance company. Consider as one alternative an option exchange which creates a market for put options on municipal bonds. Investors could then achieve the same protection against loss by buying an 'uninsured' municipal bond and a put option on that bond. (With a standard fixed exercise price, the put would actually provide more protection because it covers losses in the value of the bond for any reason, not just issuer default. However, the coverage could effectively be narrowed to only default risk by making the exercise price float to equal the current price of an AAA bond with terms comparable with those of the covered bond.) Note that both structures serve the same function for investors: protection against loss from default. However, the institutions are entirely different: an options exchange is not an insurance company. Furthermore, the put

option traded on the exchange is a different product from the insurance guarantee. Nevertheless, although the products and institutions that provide them are both quite different, the economic function they serve is the same.

In certain environments, it is surely possible that an options exchange with market-to-market collateral and a clearing corporation could be a 'better credit' than an insurance company and thereby, also be a superior institutional structure to serve the guarantee function. In such environments, the institutionally oriented manager may miss recognizing the firm's prime competitor. Regulatory bodies for financial services are almost exclusively organized along institutional lines and therefore, they face similar problems. Because options are not insurance products and exchanges are not insurance companies, insurance regulators would have no control over the option exchange even though its product is a perfect substitute for an insurance product.

The increasing flexibility and global mobility of financial institutions, together with the technology for creating custom financial contracts at low cost, have far-reaching implications not only for the regulation of financial services, but for national monetary and fiscal policies as well. Thus, policy-makers are effectively speculating against a long-run trend of declining transactions costs if they assume that traditional frictions within their individual financial systems will continue to allow national governments to pursue monetary and related financial policies with the same degree of control as in the past. If, for instance, the principal mechanism of central bank influence on macroinvestment policy comes from controlling banks that ration credit to their customers (Stiglitz & Weiss 1981), then what would be the impact on that mechanism from the creation of a national mid-market lending market similar to the US mortgage market (Cushman 1993)? Much the same point applies to a nation's fiscal policy which will surely be further constrained not just with respect to taxes targeted on financial services and transactions, but even with respect to general personal and corporate income taxes. (See Lindgren & Westlund (1991) and Umlauf (1993) on the recent Swedish experience with a transaction tax.) Scholes & Wolfson (1992) provide a general development of the theory and application of financial instruments and alternative institutional designs to respond to differing tax and regulatory structures. These techniques have a greatly magnified influence in a low-transaction-cost and global environment.

Risk management is perhaps the central topic for the management of financial institutions in the 1990s. (The 1993 survey on international banking in the *Economist* is devoted entirely to risk management of banks and its implications for bank managers and bank regulators in the future (Freeman 1993). Bankers Trust has been identified as perhaps the prime instance of a major global financial institution that has built its corporate strategy and operations around a mathematically sophisticated risk-management system (see Loomis 1992; Picker 1992; Sanford 1993).) Of course, risk management has always been important to such institutions. However, the focus in the past has been on capital adequacy to ensure performance. Equity capital is a robust all-purpose cushion against unanticipated losses, though at times a quite expensive and inefficient means for doing so. With the vast array of financial instruments and quantitative models for estimating exposures to risk, there is now a greater opportunity to eliminate risk exposures of the firm on a more targeted and efficient basis by hedging specific, non-value-enhancing risks. The cost is that the user of hedging techniques must have a more precise, quantitative assessment of the firm's business risks than the user of equity capital. In turn, the

greater need for precision places greater demands on the use and accuracy of mathematical models that measure exposures. With the spread of non-financial firms' use of the hedging alternative to equity capital for managing risk, further growth in the application of these mathematical models is likely in the future.

Institutional investment managers are generally classified by categories in terms of the assets managed: e.g. 'U.S. equities' or 'U.S. corporate debt'. However, with the development of low-cost derivative contracts, these classifications are rapidly losing any meaning. For example, a superior-performing manager in the fixed-income area can be transformed into a superior-performing manager of equities by the use of a swap contract. Suppose a bond manager can outperform standard bond-index returns by 2% (200 basis points) on average. Suppose, however, that an institutional investor is looking for a superior equity manager. If the bond manager places the investor's funds in the bond portfolio and simultaneously, enters into a swap agreement with a third party to pay the returns on the standard bond index and receive the returns on the standard equity index (for instance, in the U.S., the Standard & Poor's 500; up to transaction fees, by arbitrage, the cost of this swap is zero), then the (expected) net return to the investor is (bond index + 2%) - bond index + stock index = stock index + 2%. Thus, the superior bond manager is now a superior equity manager. This example shows the growing arbitrariness of traditional institutional categories for investment managers. It also illustrates a more general point, namely, the economic distinctions between institutionally defined debt and equity are becoming increasingly blurred.

This same blurring also occurs among other asset classes. The Bank for International Standards (BIS) international capital requirements for banks set a schedule of capital based on asset-class risk. For instance, US Treasury bonds are in a class that requires zero capital and mortgages are in a class that requires 4% capital. A bank that invests in US Treasury bonds and enters into an amortizing swap in which it pays the total return on those bonds and receives the total payments on mortgages will earn the economic equivalent of holding mortgages directly, but will face a BIS capital requirement much closer to zero than 4%.

All of this suggests that the future will require major revisions in the accounting conventions used in contract enforcement and implementation of regulations. Although it is too early to know what form these revisions will take, I conjecture that the new categories will be defined in terms of equivalent exposures, very much like the 'deltas' of contingent-claim securities in CCA. If this is the case, mathematical models of finance will have a new and enormous area of application. (See Sanford (1993) for a practising Chief Executive Officer's vision of the future financial marketplace and his characterization of the new classification categories in terms of 'particle finance'.)

In summary, in the vast bulk of the past, mathematical models have had a limited and ancillary impact on finance practice. But during the last two decades, these models have become central to practitioners in financial institutions and markets around the world. In the future, mathematical models are likely to have an indispensable role in the functioning of the global financial system including regulatory and accounting activities.

Any virtue can become a vice if taken to extreme, and just so with the application of mathematical models in finance practice. I therefore close with an added word of caution about their use. At times the mathematics of the models become too interesting and we lose sight of the models' ultimate purpose. The mathematics of the

models are precise, but the models are not, being only approximations to the complex, real world. Their accuracy as a useful approximation to that world varies considerably across time and place. The practitioner should therefore apply the models only tentatively, assessing their limitations carefully in each application.

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Discussion

R. V. SIMONS (*EDS-Scicon Ltd, U.K.*). Rapid change at financial institutions and further erosion of their role in favour of contracts between parties to perform specific functions is fine when the parties actually perform their contracts, but what about counterparty risk?

R. C. MERTON. Effective management of contract-default risk is critical to the continuing gains in economic efficiency from financial innovation. Although surely a complex problem, controlling counterparty risk is as surely within the available technological skills and experience of the financial community. Recent evidence that concern about contract default is influencing financial behaviour, includes the significant resources spent by financial institutions (with less than the highest credit rating) to create AAA subsidiaries for performing their contracting business.

A. D. WILKIE (*Watsons, London, U.K.*). It seems that writing options is very similar to the business of insurance. Insurance companies are required to hold additional solvency reserves in respect of their liabilities. Yet banks and market makers writing options appear to be able to do this with, relatively speaking, much smaller solvency requirements. Is this satisfactory?

R. C. MERTON. Writing options is similar to the business of insurance. However, as with traditional insurance, the level of risk exposure varies considerably across different types of options. For example, options written on a listed exchange are secured by mark-to-market collateral and a clearing corporation. The risks of options on stocks and bonds can be hedged by the purchase of the underlying stock or other securities of the firm. The ability to observe continuous prices for the option liabilities and to hedge those liabilities in the market are not typically available for traditional insurance risks; hence, the greater reliance on larger reserves. Although each case needs detailed investigation, in general banks and market makers are not under-reserved in their option-writing activities.